Forward Dynamics Equations for the Caterpillar CAT 262 Skid-Steer Loader with Wheel-Ground Interactions*

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1. Model Parameters

This document presents the explicit equations for the forward dynamics of the Caterpillar® CAT 262 Skid-Steer Loader shown in Fig. 1. These equations were obtained from the general model for skid-steer mobile manipulators (SSMM) with wheel ground interactions developed by the authors in [1, 2], where the reader can find mode details.

The Cat 262C compact skid-steer loader is modeled as a SSMM considering a floating base with four wheels and a one-DOF arm. As shown in Figs. 1 and 2 of [1], the mobile base is assigned body number 1, the wheels are bodies 2, 3, 4, 5 and the arm is the sixth body. The arm of the Cat 262C is implemented as a one-DOF rotary joint with axis-y parallel to $\boldsymbol{y}_1$. Extra bodies and joints can be easily added to represent the motion of the loader's bucket.

The specific values for the geometric and inertial parameters in Table I of [1] corresponding to the Cat 262C model are summarized in Table 1. It is to be noted that the joint location parameter h of the loader arm corresponding to frame $\mathcal{F}_6$ in Fig. 1shown in Table 1 is negative. This is because the Cat 262C arm is positioned at the rear-end of the machine, opposite to the

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Figure 1: Compact skid-steer loader at the experiment site with unloaded bucket (left) and loaded bucket (right).

front location of the arm in the general SSMM model shown in Fig. 1. The terramechanical model parameters for the elastic and damping force (see the contact force model in [3]) were set to $K = 6.89 \cdot 10^9$ N/m$^{3/2}$ and $D = 10^3$ N·s/m$^{3/2}$, respectively. These values are obtained from standard pavement and rubber wheel stiffness and damping values found in [4, 5].

Table 1: Cat 262C Model Parameters in SI Units.

Description		Mobile Base	Wheels	Manipulator
	x	$a = 3$	$2r = 0.9$	$\epsilon = 0.15$
Dimensions	y	$b = 1.6$	$w = 0.25$	$\epsilon = 0.15$
	z	$c = 1.2$	$2r = 0.9$	$l_6 = 3.3$
Mass		$m_1 = 2389$	$m_{2,3,4,5} = 47.7$	$m_6 = 1034$
Joint			$d = 0.2$	
Location		–	$e = 1$	$h = -1.2$
Parameters			$f = 0.5$	

Once each body is declared, the ground contact points are defined with respect to each body's frame $\mathcal{F}_i$ in 3D coordinates. More specifically, eight CPs are defined at each corner of the mobile base represented by a rectangular box, one CP is defined at the end of the manipulator arm where the tip of the bucket is located, and each of the four wheels has 32 CPs distributed around its perimeter. Such number of contact points on the wheels was chosen to obtain a more realistic simulation of the ground-wheel interaction.

Both the model and experimental data collected during the field tests have been made publicly available at http://ral.ing.puc.cl/ssmm.htm.

2. Explicit Forward Dynamics Equations

The inertial acceleration $\mathbf{a}_1$ of the base can be split, as shown in eq. (6) of [2], into an acceleration arising from the velocity-product terms $\mathbf{a}_{1c}$, and accelerations due to the external forces on the arm and the base $\mathbf{a}_{1/6ext}$ and $\mathbf{a}_{1ext}$, respectively. Using the notation $c_i = \cos(q_i)$, $s_i = \sin(q_i)$, and substituting the geometry and inertia parameters of Table I of [1] into eq. (6) of [2], the angular and linear components of the acceleration due to the velocity-product terms about/along the axes $\boldsymbol{x}_1$, $\boldsymbol{y}_1$ and $\boldsymbol{z}_1$ of the base coordinate frame $\mathcal{F}_1$ are:

$$
\begin{aligned}
\alpha_{1cx} = \Big(& c_6^2 l_6^2 m_1 m_6 {\omega_1}_z {I_y}_6 {\omega_1}_y + c_6^2 l_6^2 m_1 m_6 {\omega_1}_z {I_y}_6 \dot{q}_6 \\
& -2\, m_1 {I_z}_1 m_6 l_6^2 c_6^2 {\omega_1}_z \dot{q}_6 - 2\, {\omega_1}_z {\omega_1}_y c_6^2 m_1 {I_z}_1 m_6 l_6^2 \\
& + c_6^2 m_1 m_6 l_6^2 {\omega_1}_z {\omega_1}_y {I_y}_1 - m_1 m_6 l_6^2 c_6 s_6 {\omega_1}_y {\omega_1}_x {I_x}_1 \\
& + c_6 l_6^2 m_6 m_1 s_6 {\omega_1}_x {I_y}_6 {\omega_1}_y + c_6 l_6^2 m_6 s_6 m_1 {\omega_1}_x {I_y}_6 \dot{q}_6 \\
& - c_6 l_6^2 m_6 m_1 {I_z}_1 s_6 {\omega_1}_x {\omega_1}_y - 2\, c_6 {l_6}^2 m_6 m_1 {I_z}_1 s_6 {\omega_1}_x \dot{q}_6 \\
& + c_6 l_6^2 m_6 m_1 s_6 {\omega_1}_y {\omega_1}_x {I_y}_1 + m_1 {I_z}_1 {\omega_1}_z m_6 l_6^2 {\omega_1}_y \\
& +2\, m_1 {I_z}_1 {\omega_1}_z m_6 l_6^2 \dot{q}_6 + 4\, {I_z}_1 m_6 {\omega_1}_z {I_y}_6 {\omega_1}_y + 4\, {I_z}_1 m_6 {\omega_1}_z {I_y}_6 \dot{q}_6 \\
& -4\, {\omega_1}_z {\omega_1}_y {I_z}_1^2 m_6 + 4\, {\omega_1}_z {\omega_1}_y {I_z}_1 m_6 {I_y}_1 - 4\, {\omega_1}_z {\omega_1}_y m_1 {I_z}_1^2 \\
& +4\, {\omega_1}_z {\omega_1}_y m_1 {I_z}_1 {I_y}_1 + 4\, m_1 {I_z}_1 {\omega_1}_z {I_y}_6 {\omega_1}_y + 4\, m_1 {I_z}_1 {\omega_1}_z {I_y}_6 \dot{q}_6 \Big) \Big/ \\
& \Big(- c_6^2 m_1 {I_z}_1 m_6 l_6^2 + m_1 c_6^2 m_6 l_6^2 {I_x}_1 + m_1 {I_z}_1 m_6 l_6^2 \\
& +4\, {I_z}_1 {I_x}_1 m_6 + 4\, m_1 {I_x}_1 {I_z}_1 \Big)
\end{aligned}
$$

$$
\alpha_{1cy} = -\Big({\omega_1}_z {I_x}_1 {\omega_1}_x - {\omega_1}_x {I_z}_1 {\omega_1}_z + \tau_1\Big) \Big/ \left({I_y}_1\right)
$$

$$
\begin{aligned}
\alpha_{1cz} = \Big(& -2\, m_1 I_{x1} c_6 s_6 \omega_{1z} m_6 l_6^2 \dot{q}_6 - m_1 I_{x1} c_6 s_6 \omega_{1z} m_6 l_6^2 \omega_{1y} \\
& + m_1 m_6 l_6^2 c_6 s_6 \omega_{1z} \omega_{1y} I_{y1} - m_1 m_6 l_6^2 c_6 s_6 \omega_{1z} \omega_{1y} I_{z1} \\
& + s_6 m_1 m_6 {l_6}^2 c_6 \omega_{1z} I_{y6} \dot{q}_6 + s_6 m_1 m_6 l_6^2 c_6 \omega_{1z} I_{y6} \omega_{1y} \\
& + 4\, m_1 I_{x1} \omega_{1x} I_{y6} \omega_{1y} + 4\, m_1 I_{x1} \omega_{1x} I_{y6} \dot{q}_6 + m_1 m_6 l_6^2 \omega_{1x} I_{y6} \omega_{1y} \\
& + m_1 m_6 l_6^2 \omega_{1x} I_{y6} \dot{q}_6 + 2\, \omega_{1y} \omega_{1x} m_1 c_6^2 m_6 l_6^2 I_{x1} \\
& + 2\, m_1 I_{x1} c_6^2 \omega_{1x} m_6 l_6^2 \dot{q}_6 - \omega_{1y} \omega_{1x} m_1 c_6^2 m_6 l_6^2 I_{y1} \\
& - 4\, \omega_{1y} \omega_{1x} m_1 I_{x1}^2 - 4\, \omega_{1y} \omega_{1x} m_6 {I_{x1}}^2 + 4\, \omega_{1y} \omega_{1x} m_1 I_{x1} I_{y1} \\
& - \omega_{1y} \omega_{1x} m_1 m_6 l_6^2 I_{x1} + \omega_{1y} \omega_{1x} m_1 m_6 l_6^2 I_{y1} \\
& + 4\, \omega_{1y} \omega_{1x} m_6 I_{x1} I_{y1} + 4\, m_6 I_{x1} \omega_{1x} I_{y6} \omega_{1y} \\
& + 4\, m_6 I_{x1} \omega_{1x} I_{y6} \dot{q}_6 - m_1 m_6 {l_6}^2 c_6^2 \omega_{1x} I_{y6} \omega_{1y} \\
& - m_1 m_6 l_6^2 c_6^2 \omega_{1x} I_{y6} \dot{q}_6 \Big) \Big/ \Big(c_6^2 m_1 I_{z1} m_6 l_6^2 - m_1 c_6^2 m_6 l_6^2 I_{x1} \\
& - m_1 I_{z1} m_6 l_6^2 - 4\, I_{z1} I_{x1} m_6 - 4\, m_1 I_{x1} I_{z1} \Big)
\end{aligned}
$$

$$
\begin{aligned}
a_{1cx} = -\frac{1}{2} \Big(& 8\, I_{y6} m_1^2 \omega_{1y} v_{1z} - 16\, I_{y6} m_6 m_1 \omega_{1z} v_{1y} - 8\, I_{y6} m_6^2 v_{1y} \omega_{1z} \\
& + 8\, I_{y6} m_6^2 \omega_{1y} v_{1z} - 8\, I_{y6} m_1^2 \omega_{1z} v_{1y} + 2\, m_1^2 m_6 l_6^2 \omega_{1y} v_{1z} \\
& + 2\, m_6^2 l_6^2 m_1 \omega_{1y} v_{1z} - 2\, m_1^2 m_6 l_6^2 \omega_{1z} v_{1y} + 16\, I_{y6} m_6 m_1 \omega_{1y} v_{1z} \\
& - 2\, m_6^2 l_6^2 m_1 \omega_{1z} v_{1y} - 4\, c_6 I_{y6} l_6 m_6^2 {\omega_{1z}}^2 - 4\, l_6 s_6 \tau_1 m_6^2 \\
& - 4\, c_6 I_{y6} l_6 m_6^2 {\omega_{1y}}^2 - 8\, c_6 I_{y6} l_6 m_6^2 \omega_{1y} \dot{q}_6 - 4\, c_6 I_{y6} l_6 m_6^2 \dot{q}_6^2 \\
& - 4\, c_6 I_{y6} l_6 m_6 m_1 \dot{q}_6^2 - 4\, c_6 I_{y6} l_6 m_6 m_1 {\omega_{1z}}^2 - c_6 {l_6}^3 {m_6}^2 m_1 {\omega_{1x}}^2 \\
& - 4\, l_6 s_6 m_1 \tau_1 m_6 - 4\, I_{y6} l_6 s_6 m_1 \omega_{1z} \omega_{1x} m_6 + c_6^3 {l_6}^3 m_6^2 m_1 {\omega_{1x}}^2 \\
& - 2\, s_6 m_1 \omega_{1z} \omega_{1x} m_6^2 {l_6}^3 c_6^2 - 4\, c_6 I_{y6} l_6 m_6 m_1 {\omega_{1y}}^2 \\
& - 8\, c_6 I_{y6} l_6 m_6 m_1 \omega_{1y} \dot{q}_6 - 2\, c_6 {l_6}^3 m_6^2 m_1 \omega_{1y} \dot{q}_6 - c_6 {l_6}^3 m_6^2 m_1 \dot{q}_6^2 \\
& - c_6 {l_6}^3 m_6^2 m_1 {\omega_{1y}}^2 - 4\, I_{y6} l_6 s_6 \omega_{1z} \omega_{1x} m_6^2 \\
& - c_6^3 {l_6}^3 m_6^2 m_1 {\omega_{1z}}^2 \Big) \Big/ \Big(4\, I_{y6} m_1^2 + 4\, I_{y6} {m_6}^2 + 8\, I_{y6} m_1 m_6 \\
& + m_1 m_6^2 l_6^2 + m_1^2 m_6 l_6^2 \Big)
\end{aligned}
$$

$$
\begin{aligned}
a_{1cy} = \Big(& -2\, I_{z1} m_6 l_6 s_6 \omega_{1z} I_{y6} \omega_{1y} + 4\, I_{z1} I_{x1} m_6 l_6 s_6 \omega_{1z} \dot{q}_6 \\
& - 2\, I_{z1} m_6 l_6 s_6 \omega_{1z} \omega_{1y} I_{y1} + 2\, I_{z1} I_{x1} m_6 l_6 s_6 \omega_{1z} \omega_{1y} \\
& + 2\, {I_{z1}}^2 m_6 l_6 s_6 \omega_{1z} \omega_{1y} - 2\, I_{z1} m_6 l_6 s_6 \omega_{1z} I_{y6} \dot{q}_6 \\
& + 4\, m_1 I_{z1} I_{x1} \omega_{1x} v_{1z} - 4\, m_1 I_{z1} I_{x1} \omega_{1z} v_{1x} + 2\, c_6 m_6 l_6 I_{x1} \omega_{1x} I_{y6} \dot{q}_6 \\
& + 2\, c_6 m_6 l_6 I_{x1} \omega_{1x} I_{y6} \omega_{1y} - 2\, c_6 m_6 l_6 {I_{x1}}^2 \omega_{1y} \omega_{1x} \\
& - m_1 I_{z1} m_6 l_6^2 \omega_{1z} v_{1x} + m_1 I_{z1} m_6 l_6^2 \omega_{1x} v_{1z} - 4\, I_{z1} I_{x1} m_6 v_{1x} \omega_{1z} \\
& + 4\, I_{z1} I_{x1} m_6 \omega_{1x} v_{1z} - m_1 c_6^2 m_6 l_6^2 I_{x1} \omega_{1z} v_{1x} \\
& + m_1 I_{z1} c_6^2 m_6 l_6^2 \omega_{1z} v_{1x} + m_1 c_6^2 m_6 l_6^2 I_{x1} \omega_{1x} v_{1z} \\
& + 2\, c_6 m_6 l_6 I_{x1} \omega_{1y} \omega_{1x} I_{y1} - m_1 I_{z1} \left(c_6\right)^2 m_6 l_6^2 \omega_{1x} v_{1z} \\
& - 2\, I_{z1} I_{x1} m_6 l_6 c_6 \omega_{1x} \omega_{1y} - 4\, I_{z1} I_{x1} m_6 l_6 c_6 \omega_{1x} \dot{q}_6 \Big) \Big/ \\
& \Big(-c_6^2 m_1 I_{z1} m_6 {l_6}^2 + m_1 c_6^2 m_6 l_6^2 I_{x1} + m_1 I_{z1} m_6 l_6^2 \\
& + 4\, I_{z1} I_{x1} m_6 + 4\, m_1 I_{x1} I_{z1} \Big)
\end{aligned}
$$

$$
\begin{aligned}
a_{1cz} = \frac{1}{2}\Big(& m_1 m_6^2 {l_6}^3 c_6^2 {\omega_1}_x^2 s_6 - 2\, m_1 m_6^2 {l_6}^3 \omega_{1x} c_6 \omega_{1z} \\
&+4\, m_1 c_6 m_6 l_6 \tau_1 + 8\, I_{y6} v_{1x} \omega_{1y} m_1^2 - 8\, I_{y6} m_1^2 \omega_{1x} v_{1y} \\
&+8\, I_{y6} m_6^2 \omega_{1y} v_{1x} - 2\, m_6^2 m_1 l_6^2 \omega_{1x} v_{1y} + 2\, m_6^2 m_1 l_6^2 \omega_{1y} v_{1x} \\
&+16\, I_{y6} m_6 m_1 \omega_{1y} v_{1x} - 16\, I_{y6} m_6 m_1 \omega_{1x} v_{1y} \\
&+2\, m_1 m_6^2 {l_6}^3 c_6^3 \omega_{1x} \omega_{1z} + 2\, m_1^2 m_6 l_6^2 \omega_{1y} v_{1x} \\
&-2\, m_1^2 m_6 l_6^2 \omega_{1x} v_{1y} - 8\, I_{y6} m_6^2 v_{1y} \omega_{1x} - 2\, m_1 s_6 m_6^2 {l_6}^3 \omega_{1y} \dot{q}_6 \\
&-m_1 s_6 m_6^2 {l_6}^3 \dot{q}_6^2 - m_1 s_6 m_6^2 {l_6}^3 {\omega_1}_y^2 - s_6 m_1 m_6^2 {l_6}^3 {\omega_1}_x^2 \\
&-m_1 s_6 m_6^2 {l_6}^3 \left(c_6\right)^2 {\omega_1}_z^2 - 4\, I_{y6} m_6^2 s_6 l_6 \dot{q}_6^2 \\
&-8\, I_{y6} m_6^2 s_6 l_6 \omega_{1y} \dot{q}_6 - 4\, I_{y6} m_6^2 l_6 \omega_{1x} c_6 \omega_{1z} - 4\, I_{y6} m_1 s_6 m_6 l_6 \dot{q}_6^2 \\
&-4\, I_{y6} m_6^2 s_6 l_6 {\omega_1}_y^2 - 8\, I_{y6} m_1 s_6 m_6 l_6 \omega_{1y} \dot{q}_6 - 4\, I_{y6} s_6 m_1 m_6 l_6 {\omega_1}_x^2 \\
&-4\, I_{y6} s_6 m_6^2 l_6 {\omega_1}_x^2 - 4\, I_{y6} m_1 m_6 l_6 \omega_{1x} c_6 \omega_{1z} + 4\, {m_6}^2 c_6 l_6 \tau_1\Big) \Big/ \\
&\left(4\, I_{y6} m_1^2 + 4\, I_{y6} m_6^2 + 8\, I_{y6} m_1 m_6 + m_1 m_6^2 l_6^2 + m_1^2 m_6 {l_6}^2\right)
\end{aligned}
$$

The angular and linear components of the base acceleration due to the external forces on the arm $\mathbf{a}_{1/6ext}$ in the coordinates of $\mathcal{F}_1$ are given by:

$$
\begin{aligned}
\alpha_{1/6ext_x} = \Big(& 4\, c_6 m_1 I_{z1} n_{6ext\,x} + 4\, c_6 I_{z1} n_{6ext\,x}\, m_6 \\
&+4\, m_1 I_{z1} s_6 n_{6ext\,z} - 2\, l_6 m_6 I_{z1} s_6 f_{6ext\,y} + 4\, m_6 s_6 I_{z1} n_{6ext\,z} \\
&+c_6 l_6^2 m_6 m_1 n_{6ext\,x}\Big) \Big/ \Big(- c_6^2 m_1 I_{z1} m_6 l_6^2 + m_1 c_6^2 m_6 l_6^2 I_{x1} \\
&+m_1 I_{z1} m_6 l_6^2 + 4\, I_{z1} I_{x1} m_6 + 4\, m_1 I_{x1} I_{z1}\Big)
\end{aligned}
$$

$$
\alpha_{1/6ext_y} = 0
$$

$$
\begin{aligned}
\alpha_{1/6ext_z} = -\Big(& m_1 m_6 l_6^2 s_6 n_{6ext\,x} + 2\, c_6 m_6 l_6 I_{x1} f_{6ext\,y} \\
&-4\, m_1 I_{x1} c_6 n_{6ext\,z} + 4\, I_{x1} m_6 s_6 n_{6ext\,x} - 4\, I_{x1} m_6 c_6 n_{6ext\,z} \\
&+4\, m_1 I_{x1} s_6 n_{6ext\,x}\Big) \Big/ \Big(- c_6^2 m_1 I_{z1} m_6 l_6^2 + m_1 c_6^2 m_6 l_6^2 I_{x1} \\
&+m_1 I_{z1} m_6 l_6^2 + 4\, I_{z1} I_{x1} m_6 + 4\, m_1 I_{x1} I_{z1}\Big)
\end{aligned}
$$

$$
\begin{aligned}
a_{1/6ext_x} = \Big(& l_6^2 m_1 m_6 s_6 f_{6ext\,z} + 4\, c_6 I_{y6} m_1 f_{6ext\,x} + 4\, I_{y6} m_6 s_6 f_{6ext\,z} \\
&+2\, l_6 m_6 m_1 s_6 n_{6ext\,y} + 2\, m_6^2 s_6 l_6 n_{6ext\,y} + 4\, I_{y6} m_1 s_6 f_{6ext\,z} \\
&+4\, c_6 I_{y6} m_6 f_{6ext\,x} + m_6^2 s_6 f_{6ext\,z}\, l_6^2 + c_6 m_1 m_6 l_6^2 f_{6ext\,x}\Big) \Big/ \\
&\Big(4\, I_{y6} m_1^2 + 4\, I_{y6} m_6^2 + 8\, I_{y6} m_1 m_6 + m_1 m_6^2 l_6^2 + m_1^2 m_6 l_6^2\Big)
\end{aligned}
$$

$$
\begin{aligned}
a_{1/6ext_y} = \Big(& - 2\, I_{z1} m_6 l_6 n_{6ext\,z} + I_{z1} m_6 l_6^2 f_{6ext\,y} + c_6^2 m_6 l_6^2 I_{x1} f_{6ext\,y} \\
&-2\, I_{z1} m_6 l_6 s_6 c_6 n_{6ext\,x} + 2\, c_6^2 I_{z1} m_6 l_6 n_{6ext\,z} \\
&+2\, c_6 m_6 l_6 I_{x1} s_6 n_{6ext\,x} - 2\, c_6^2 m_6 l_6 I_{x1} n_{6ext\,z} \\
&-I_{z1} c_6^2 m_6 l_6^2 f_{6ext\,y} + 4\, I_{z1} I_{x1} f_{6ext\,y}\Big) \Big/ \Big(- c_6^2 m_1 I_{z1} m_6 l_6^2 \\
&+m_1 c_6^2 m_6 l_6^2 I_{x1} + m_1 I_{z1} m_6 l_6^2 + 4\, I_{z1} I_{x1} m_6 + 4\, m_1 I_{x1} I_{z1}\Big)
\end{aligned}
$$

$$
\begin{aligned}
a_{1/6ext_z} = -\Big(& - m_1 c_6 f_{6ext\,z}\, m_6 l_6^2 - 2\, m_6^2 c_6 l_6 n_{6ext\,y} \\
& +4\, I_{y\,6} m_6 s_6 f_{6ext\,x} - 4\, I_{y\,6} m_1 c_6 f_{6ext\,z} + 4\, I_{y\,6} m_1 s_6 f_{6ext\,x} \\
& -4\, I_{y\,6} m_6 c_6 f_{6ext\,z} - m_6^2 c_6 f_{6ext\,z}\, l_6^2 + m_1 s_6 f_{6ext\,x}\, m_6 l_6^2 \\
& -2\, m_1 c_6 m_6 l_6 n_{6ext\,y} \Big) \Big/ \Big(4\, I_{y\,6} m_1^2 + 4\, I_{y\,6} m_6^2 \\
& +8\, I_{y\,6} m_1 m_6 + m_1 m_6^2 l_6^2 + m_1^2 m_6 l_6^2 \Big)
\end{aligned}
$$

Finally, the angular and linear components of the base acceleration due to the external forces on the base $\mathbf{a}_{1ext}$ in the coordinates of $\mathcal{F}_1$ are given by:

$$
\begin{aligned}
\alpha_{1ext\,x} = \Big(& 4\, m_6 I_{z\,1} n_{1ext\,x} - c_6 l_6^2 m_6 m_1 s_6 n_{1ext\,z} - 2\, l_6 m_6 I_{z\,1} s_6 f_{1ext\,y} \\
& +4\, m_1 I_{z\,1} n_{1ext\,x} + c_6^2 l_6^2 m_6 m_1 n_{1ext\,x} \Big) \Big/ \Big(- c_6^2 m_1 I_{z\,1} m_6 l_6^2 \\
& +m_1 c_6^2 m_6 l_6^2 I_{x\,1} + m_1 I_{z\,1} m_6 l_6^2 + 4\, I_{z\,1} I_{x\,1} m_6 + 4\, m_1 I_{x\,1} I_{z\,1} \Big)
\end{aligned}
$$

$$
\alpha_{1ext\,y} = n_{1ext\,y} / I_{y\,1}
$$

$$
\begin{aligned}
\alpha_{1ext\,z} = \Big(& 4\, I_{x\,1} m_6 n_{1ext\,z} + 4\, m_1 I_{x\,1} n_{1ext\,z} + m_1 m_6 l_6^2 n_{1ext\,z} \\
& -m_1 m_6 l_6^2 \left(c_6\right)^2 n_{1ext\,z} - 2\, c_6 m_6 l_6 I_{x\,1} f_{1ext\,y} \\
& -m_1 m_6 l_6^2 s_6 c_6 n_{1ext\,x} \Big) \Big/ \Big(- c_6^2 m_1 I_{z\,1} m_6 l_6^2 \\
& +m_1 c_6^2 m_6 l_6^2 I_{x\,1} + m_1 I_{z\,1} m_6 l_6^2 + 4\, I_{z\,1} I_{x\,1} m_6 + 4\, m_1 I_{x\,1} I_{z\,1} \Big)
\end{aligned}
$$

$$
\begin{aligned}
a_{1ext\,x} = -\Big(& - m_6^2 f_{1ext\,x}\, l_6^2 - c_6 m_6^2 s_6 l_6^2 f_{1ext\,z} + c_6^2 m_6^2 f_{1ext\,x}\, l_6^2 \\
& -l_6^2 m_1 m_6 f_{1ext\,x} - 4\, I_{y\,6} f_{1ext\,x}\, m_1 - 4\, I_{y\,6} m_6 f_{1ext\,x} \Big) \Big/ \\
& \Big(4\, I_{y\,6} m_1^2 + 4\, I_{y\,6} m_6^2 + 8\, I_{y\,6} m_1 m_6 + m_1 m_6^2 l_6^2 + m_1^2 m_6 l_6^2 \Big)
\end{aligned}
$$

$$
\begin{aligned}
a_{1ext\,y} = -\Big(& I_{z\,1} c_6^2 m_6 l_6^2 f_{1ext\,y} + 2\, c_6 m_6 l_6 I_{x\,1} n_{1ext\,z} \\
& -I_{z\,1} m_6 l_6^2 f_{1ext\,y} - c_6^2 m_6 l_6^2 I_{x\,1} f_{1ext\,y} + 2\, I_{z\,1} m_6 l_6 s_6 n_{1ext\,x} \\
& -4\, I_{z\,1} I_{x\,1} f_{1ext\,y} \Big) \Big/ \Big(- c_6^2 m_1 I_{z\,1} m_6 l_6^2 + m_1 c_6^2 m_6 l_6^2 I_{x\,1} \\
& +m_1 I_{z\,1} m_6 l_6^2 + 4\, I_{z\,1} I_{x\,1} m_6 + 4\, m_1 I_{x\,1} I_{z\,1} \Big)
\end{aligned}
$$

$$
\begin{aligned}
a_{1ext\,z} = \Big(& m_1 f_{1ext\,z}\, m_6 l_6^2 + \left(c_6\right)^2 m_6^2 l_6^2 f_{1ext\,z} + 4\, I_{y\,6} m_6 f_{1ext\,z} \\
& +4\, I_{y\,6} f_{1ext\,z}\, m_1 + c_6 m_6^2 s_6 l_6^2 f_{1ext\,x} \Big) \Big/ \Big(4\, I_{y\,6} m_1^2 + 4\, I_{y\,6} m_6^2 \\
& +8\, I_{y\,6} m_1 m_6 + m_1 m_6^2 l_6^2 + m_1^2 m_6 l_6^2 \Big)
\end{aligned}
$$

In an analogous way to the computation of the base acceleration, the joint acceleration $\ddot{q}_6$ in (**??**) can be split into a term associated to the velocity-product terms $\ddot{q}_{6c}$ that does not include the external forces, and two other terms that include the external forces acting on the arm and the base, $\ddot{q}_{6ext}$ and $\ddot{q}_{6/1ext}$, respectively. These terms are:

$$
\begin{aligned}
\ddot{q}_{6c} = \Big(& 4\, I_{y_6} m_1 \omega_{1z} I_{x1} \omega_{1x} - 4\, I_{y_6} m_1 \omega_{1x} I_{z1} \omega_{1z} - m_1 \omega_{1x} I_{z1} \omega_{1z} m_6 l_6^2 \\
& - m_1 m_6 l_6^2 I_{y_1} \omega_{1z} \omega_{1x} + 2\, m_1 m_6 {l_6}^2 I_{y_1} c_1^2 \omega_{1x} \omega_{1z} \\
& - m_1 m_6 l_6^2 I_{y_1} s_1 {\omega_{1z}}^2 c_1 + m_1 \omega_{1z} I_{x1} \omega_{1x} m_6 l_6^2 \\
& + m_1 m_6 l_6^2 I_{y_1} c_1 {\omega_{1x}}^2 s_1 + 4\, m_1 \tau_6 I_{y_1} + 8\, I_{y_6} \tau_6 m_1 + 2\, m_1 \tau_6 m_6 l_6^2 \\
& - 8\, I_{y_6} \omega_{1x} I_{z1} \omega_{1z} m_6 + 16\, I_{y_6} m_6 \tau_6 + 8\, I_{y_6} \omega_{1z} I_{x1} \omega_{1x} m_6 \\
& + 8\, \tau_6 I_{y_1} m_6 \Big) \Big/ \Big(\left(4\, m_1 I_{y_6} + m_1 m_6 l_1^2 + 8\, I_{y_6} m_6\right) I_{y_1} \Big), \\
\ddot{q}_{6ext} = \big(& 4\, n_{6ext\,y}\, m_1 + 4\, n_{6ext\,y}\, m_6 + 2 m_6 l_6 f_{6ext\,z} \big) \Big/ \\
& \left(4\, I_{y_6} m_1 + m_6 l_6^2 m_1 + 4\, m_6 I_{y_6}\right), \\
\ddot{q}_{6/1ext} = \Big(& -4\, m_1 I_{y_6} n_{1ext\,y} - 4\, I_{y_6} n_{1ext\,y}\, m_6 - n_{1ext\,y}\, m_6 l_6^2 m_1 \\
& + 2\, m_6 l_6 I_{y_1} c_6 f_{1ext\,z} + 2\, m_6 l_6 I_{y_1} s_6 f_{1ext\,x} \Big) \Big/ \\
& \Big(\left(4\, I_{y_6} m_1 + m_6 l_6^2 m_1 + 4\, m_6 I_{y_6}\right) I_{y_1} \Big).
\end{aligned}
$$

■

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