$\mathcal{H}_\infty$ Controller Matrices of the Terrain Disturbance Rejection Strategy for Hydraulically Actuated Mobile Manipulators with a Non-Rigid Link

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I. $\mathcal{H}_\infty$-CONTROLLER

THE $\mathcal{H}_\infty$ controller is implemented as a dynamical system:

$$\begin{aligned}\dot{\hat{\boldsymbol{x}}}_\infty &= \hat{\boldsymbol{A}}_\infty \hat{\boldsymbol{x}}_\infty + \hat{\boldsymbol{B}}_\infty \boldsymbol{y} \\ \boldsymbol{u}_\infty &= \hat{\boldsymbol{C}}_\infty \hat{\boldsymbol{x}}_\infty \end{aligned} \tag{1}$$

with input $\boldsymbol{y}$, output $\boldsymbol{u}_\infty$, internal state $\hat{\boldsymbol{x}}_\infty$ and state-space matrices $\hat{\boldsymbol{A}}_\infty, \hat{\boldsymbol{B}}_\infty, \hat{\boldsymbol{C}}_\infty$. The state-space matrices for Controllers 1, 2 and 3 are given respectively in (2), (3), (4).

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$$\hat{\boldsymbol{A}}_{\infty_1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -3.532\text{e}1 & -5.842\text{e}2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.842\text{e}2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 5.778\text{e}{-3} & -1.158\text{e}{-3} & -2.179\text{e}1 & -1.390\text{e}1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.390\text{e}1 & 0 & 0 & 0 \\ 1 & 0 & 5.778\text{e}{-3} & -1.158\text{e}{-3} & -2.137\text{e}1 & -6.226\text{e}0 & -2.652\text{e}0 & -1.139\text{e}1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.139\text{e}1 & 0 \end{bmatrix} \quad \hat{\boldsymbol{B}}_{\infty_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \hat{\boldsymbol{C}}^T_{\infty_1} = \begin{bmatrix} -6.039\text{e}5 \\ 0 \\ -3.490\text{e}3 \\ 6.993\text{e}2 \\ 1.290\text{e}7 \\ 3.760\text{e}6 \\ -2.424\text{e}6 \\ 6.877\text{e}6 \end{bmatrix}$$

(2)

$$\hat{\boldsymbol{A}}_{\infty_2} = \begin{bmatrix} -2.356\text{e}{-1} & 1.614\text{e}{-5} & -2.356\text{e}{-1} & 0 & -2.356\text{e}{-1} & 0 & 0 & 0 \\ 7.943\text{e}2 & -7.960\text{e}3 & 7.943\text{e}2 & 0 & 7.943\text{e}2 & 0 & 0 & 0 \\ -3.680\text{e}{-1} & 3.481\text{e}{-1} & -3.680\text{e}{-1} & 1 & -3.680\text{e}{-1} & 0 & 0 & 0 \\ -2.770\text{e}2 & 2.772\text{e}3 & -6.399\text{e}2 & -1.458\text{e}0 & -2.732\text{e}2 & 4.209\text{e}0 & -2.816\text{e}{-5} & 0 \\ -5.276\text{e}{-1} & 1.957\text{e}0 & -5.276\text{e}{-1} & 0 & -5.276\text{e}{-1} & 1 & 0 & 0 \\ -1.555\text{e}3 & 1.558\text{e}4 & 5.316\text{e}2 & 8.417\text{e}0 & -1.587\text{e}3 & -3.429\text{e}1 & 2.294\text{e}{-4} & 0 \\ 2.905\text{e}8 & -2.908\text{e}9 & 2.905\text{e}8 & 0 & 2.905\text{e}8 & -1.486\text{e}9 & 5.821\text{e}{-11} & 1.260\text{e}5 \\ -1.294\text{e}4 & -8.055\text{e}4 & 1.862\text{e}2 & 7.037\text{e}0 & -1.293\text{e}4 & 8.644\text{e}{-1} & -8.718\text{e}{-6} & -6.830\text{e}0 \end{bmatrix} \quad \hat{\boldsymbol{B}}_{\infty_2} = \begin{bmatrix} 8.760\text{e}{-1} & 2.064e-1 \\ 6.973\text{e}3 & -6.958\text{e}2 \\ -3.049\text{e}{-1} & 3.224\text{e}{-1} \\ -2.428\text{e}3 & 2.427\text{e}2 \\ -1.714\text{e}0 & 4.622\text{e}{-1} \\ -1.365\text{e}4 & 1.363\text{e}3 \\ 2.548\text{e}9 & -2.544\text{e}8 \\ 8.499\text{e}{-5} & 2.915\text{e}{-1} \end{bmatrix}$$

$$\hat{\boldsymbol{C}}_{\infty_2} = \begin{bmatrix} -2.215\text{e}3 & -1.379\text{e}4 & 3.193\text{e}1 & 1.205\text{e}0 & -2.215\text{e}3 & 1.480\text{e}{-1} & -1.493\text{e}{-6} & -2.788\text{e}{-2} \end{bmatrix}$$

(3)

$$\hat{\boldsymbol{A}}_{\infty_3} = \begin{bmatrix} -1.003\text{e}{-1} & 5.159\text{e}{-5} & -1.003\text{e}{-1} & 0 & -5.327\text{e}{-02} & 0 & -2.052\text{e}{-2} & 0 & 0 & 0 & 0 & 0 \\ 9.963\text{e}0 & -1.000\text{e}3 & 9.963\text{e}0 & 0 & 7.398\text{e}0 & 0 & 3.563\text{e}0 & 0 & 0 & 0 & 0 & 0 \\ 1.089\text{e}{-4} & -1.069\text{e}{-2} & 1.089\text{e}{-4} & 1 & -1.201\text{e}{-3} & 0 & -9.803\text{e}{-4} & 0 & 0 & 0 & 0 & 0 \\ 1.064\text{e}{-1} & -1.069\text{e}1 & -2.419\text{e}3 & -3.816\text{e}0 & 2.752\text{e}3 & 9.624\text{e}0 & -2.014\text{e}2 & -1.053\text{e}1 & 2.053\text{e}{-6} & 0 & 1.454\text{e}{-5} & 0 \\ -1.623\text{e}{-2} & 1.273\text{e}0 & -1.623\text{e}{-2} & 0 & -3.690\text{e}{-1} & 1 & -2.108\text{e}{-1} & 0 & 0 & 0 & 0 & 0 \\ -1.269\text{e}1 & 1.273\text{e}3 & 7.028\text{e}3 & 9.624\text{e}0 & -8.093\text{e}3 & -2.803\text{e}1 & 8.518\text{e}2 & 3.931\text{e}1 & -7.666\text{e}6 & 0 & -3.668\text{e}{-5} & 0 \\ 2.351\text{e}{-3} & -2.718\text{e}{-1} & 2.351\text{e}{-3} & 0 & -3.584\text{e}{-2} & 0 & -2.153\text{e}{-2} & 1 & 0 & 0 & 0 & 0 \\ 2.708\text{e}0 & -2.717\text{e}2 & -9.857\text{e}3 & -1.053\text{e}1 & 1.185\text{e}4 & 3.931\text{e}1 & -2.152\text{e}3 & -9.231\text{e}1 & 1.800\text{e}{-5} & 0 & 4.012\text{e}{-5} & 0 \\ -7.679\text{e}{+6} & 8.908\text{e}{+8} & -7.679\text{e}{+6} & 0 & 1.151\text{e}{+8} & 0 & 6.832\text{e}{+7} & -3.278\text{e}{+9} & 5.821\text{e}{-11} & 1.339\text{e}{+4} & 0 & 0 \\ -1.470\text{e}3 & -8.713\text{e}4 & -3.044\text{e}3 & 1.271\text{e}{-1} & 1.198\text{e}0 & 6.483\text{e}{-2} & -1.569\text{e}3 & 1.321\text{e}{-2} & -4.789\text{e}{-7} & -6.668\text{e}0 & -3.396\text{e}{-8} & -4.240\text{e}{-2} \\ -9.056\text{e}6 & 9.588\text{e}8 & -9.057\text{e}6 & -8.969\text{e}10 & 4.370\text{e}7 & 0 & 2.627\text{e}7 & 0 & 0 & 0 & 5.821\text{e}{-11} & 8.336\text{e}6 \\ -6.470\text{e}2 & -3.263\text{e}4 & -1.343\text{e}3 & 3.625\text{e}{-1} & 1.688\text{e}0 & 1.848\text{e}{-1} & -6.855\text{e}2 & 3.760\text{e}{-2} & -2.092\text{e}{-7} & -4.198\text{e}{-4} & -1.500\text{e}{-8} & -6.685\text{e}0 \end{bmatrix}$$

$$\hat{\boldsymbol{B}}^T_{\infty_3} = \begin{bmatrix} 4.313\text{e}{-1} & 4.313\text{e}2 & 4.611\text{e}{-3} & 4.610\text{e}0 & -5.490\text{e}{-1} & -5.491\text{e}2 & 1.172\text{e}{-1} & 1.172\text{e}2 & -3.842\text{e}8 & -1.001\text{e}{-8} & -4.135\text{e}8 & -1.001\text{e}{-8} \\ 2.028\text{e}{-2} & -1.106\text{e}0 & -5.649\text{e}{-4} & -1.543\text{e}{-2} & -1.522\text{e}{-1} & 1.618\text{e}0 & -1.647\text{e}{-2} & -4.095\text{e}{-1} & 5.297\text{e}7 & 1.434\text{e}{-5} & 2.275\text{e}7 & 1.434\text{e}{-5} \\ 1.413\text{e}{-2} & -1.654\text{e}0 & 9.507\text{e}{-5} & -1.532\text{e}{-2} & 6.826\text{e}{-2} & 2.010\text{e}0 & 6.171\text{e}{-3} & -3.874\text{e}{-1} & -2.019\text{e}7 & 1.396\text{e}{-5} & -7.517\text{e}6 & 1.396\text{e}{-5} \\ 8.850\text{e}{-3} & -1.537\text{e}0 & 4.228\text{e}{-4} & -1.516\text{e}{-2} & 9.090\text{e}{-2} & 1.847\text{e}0 & 9.286\text{e}{-3} & -3.710\text{e}{-1} & -2.947\text{e}7 & 1.399\text{e}{-5} & -1.133\text{e}7 & 1.399\text{e}{-5} \end{bmatrix}$$

$$\hat{\boldsymbol{C}}_{\infty_3} = \begin{bmatrix} -2.556\text{e}{+02} & -1.515\text{e}{+04} & -5.294\text{e}{+02} & 2.210\text{e}{-02} & 2.083\text{e}{-01} & 1.127\text{e}{-02} & -2.729\text{e}{+02} & 2.297\text{e}{-03} & -8.328\text{e}{-08} & -1.671\text{e}{-04} & -5.906\text{e}{-09} & -7.373\text{e}{-03} \\ -1.125\text{e}{+02} & -5.674\text{e}{+03} & -2.335\text{e}{+02} & 6.304\text{e}{-02} & 2.935\text{e}{-01} & 3.214\text{e}{-02} & -1.192\text{e}{+02} & 6.538\text{e}{-03} & -3.638\text{e}{-08} & -7.300\text{e}{-05} & -2.609\text{e}{-09} & -3.258\text{e}{-03} \end{bmatrix}$$

(4)